IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Decoherence by engineered quantum baths

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2007 J. Phys. A: Math. Theor. 40 8033

(http://iopscience.iop.org/1751-8121/40/28/S12)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.109
The article was downloaded on 03/06/2010 at 05:20

Please note that terms and conditions apply.

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **40** (2007) 8033–8040 doi:10.1088/1751-8113/40/28/S12

Decoherence by engineered quantum baths

Davide Rossini[1], **Tommaso Calarco**[2,3], **Vittorio Giovannetti**[1], **Simone Montangero**[1,4] **and Rosario Fazio**[1,5]

[1] NEST-CNR-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy
[2] Dipartimento di Fisica, Università di Trento and BEC-CNR-INFM, I-38050 Povo, Italy
[3] ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, MA 02138, USA
[4] Institut für Theoretische Festkörperphysik, Universität Karlsruhe, 76128 Germany
[5] International School for Advanced Studies (SISSA), via Beirut 2-4, I-34014 Trieste, Italy

Received 9 October 2006, in final form 11 December 2006
Published 27 June 2007
Online at stacks.iop.org/JPhysA/40/8033

Abstract
Optical lattices can be used to simulate quantum baths and hence they can be of fundamental help to study, in a controlled way, the emergence of decoherence in quantum systems. Here we show how to implement a pure dephasing model for a two-level system coupled to an interacting spin bath. In this scheme it is possible to implement a large variety of spin environments embracing Ising, XY and Heisenberg universality classes. After having introduced the model, we calculate exactly the decoherence for the Ising and the XY spin bath model. We find universal features depending on the critical behaviour of the spin bath, both in the short- and long-time limits. The rich scenario that emerges can be tested experimentally and can be of importance for the understanding of the coherence loss in open quantum systems.

PACS numbers: 03.75.Lm, 03.65.Yz, 05.50.+q

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The confinement of cold atomic gases in optical lattices has been the successful step towards the implementation of a large variety of model systems of fundamental importance in condensed matter (see [1] for a comprehensive review on this subject). Among the numerous beautiful experiments in this area we mention the first evidence of the Mott-to-Superfluid transition by Greiner *et al* [2] following the theoretical prediction made in [3]. One of the most attractive features of these systems is the possibility to control, by changing the depth of the optical potential, the couplings of the underlying model system. Moreover, by loading the lattice with different atoms, it is possible to simulate a large variety of strongly correlated systems [1]. Optical lattices are disorder free and therefore can be considered as an ideal playground for the study of a number of complex physical systems. Duan *et al* [4] and Jané *et al* [5] have

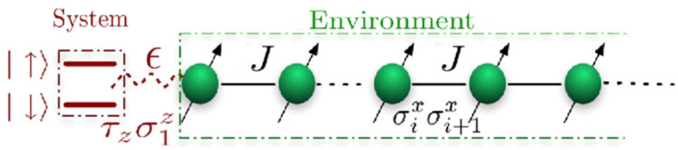

Figure 1. A sketch of the system-plus-bath model we consider in this work. The two-level system (at position zero) is coupled to the σ^z component of the first spin of the chain that acts as a spin bath. Atoms in an optical lattice can simulate this controlled decoherence by means of series of lasers and displacements of the lattice, which allow us to realize both the interaction of the bath with an external magnetic field and the anisotropic exchange coupling.

shown how to implement textbook models of magnetism by means of cold atoms in optical lattices.

In this paper we would like to suggest the use of optical lattices in another important area of physics, the study open quantum systems. Understanding decoherence is central to the description of the crossover between quantum and classical behaviour [6, 7] and important for a successful implementation of quantum information processing [8]. Unfortunately it is not always possible to fully characterize the bath and therefore it is necessary to resort to ingenious modelizations (paradigmatic models are harmonic [9] or spin [10] baths). Here we take a different point of view, i.e. we want to engineer a quantum bath. Ultracold atoms in optical lattices provide an interesting example where the properties of the bath and the system–bath interaction are controllable. This *tunable* open quantum system has several interesting aspects. First of all it can be realized experimentally. Moreover the model we propose embraces a variety of interacting spin baths and allows us to investigate a fairly rich scenario. In the case of Ising or XY bath we obtain an *exact* solution for the decoherence, expressed in terms of the temporal decay of the so-called Loschmidt echo. This quantity has been introduced in the context of the quantum chaos, to describe the hypersensitivity of the time evolution to perturbations experienced by the surrounding system [11], and then subsequently used to characterize loss of coherence for a variety of situations (see, for example [12–16]). In our case, we provide a non-trivial example where the emergence of decoherence in quantum systems can be controlled and tested experimentally.

The paper is organized as follows. In section 2 we discuss the general framework for the realization of controlled open quantum systems by using optical lattices. In section 3 we introduce the model Hamiltonian underlying this implementation and derive an analytic formula for the decay of coherence factors of the system's reduced density matrix, in the case when the bath is an Ising or an XY spin chain. In section 4 we explicitly show results concerning the loss of coherence induced by these types of environment. Both short- and long-time behaviour are considered; the scaling with the bath size is also discussed. Finally, in section 5, we draw our conclusions.

2. Simulation of open systems by optical lattices

As a prototype open quantum system we consider a two-level system (qubit) coupled to a quantum bath (see figure 1). Since we want to ignore relaxation, we will consider a pure dephasing model where the populations of the ground and excited states do not evolve in time and only the coherences will decay. The model of environment is inspired by the recent proposals to implement spin models by means of optical lattices [4, 5]; it consists in an *interacting* spin bath. It is worth stressing that a straightforward implementation of the method discussed in [4, 5] is not possible. The reason is that the properties of the quantum system should be distinct from those of the spins constituting the bath and, in addition, the interaction between the quantum system and the bath is *different* from the interaction of the spin within the bath. In order to include these two crucial ingredients we found more convenient to implement the ideas of [5] to the present case. The key advantage is that we can realize the system-plus-bath setup by using a single one-dimensional lattice in which the quantum system is placed in a given site of the lattice (for example the first one). The different Hamiltonian for the system and for the bath is realized by a different sequence of pulses that simulate the dynamics of the model. The same holds for the different coupling Hamiltonian of the two-level system with the bath and the couplings within the bath. The left-most atom simulates the two-level system, the coupling to the second site is the interaction between the quantum system and the environment, the rest of the chain is the interacting spin environment[6].

Jané *et al* showed that atoms loaded in an optical lattice can simulate the evolution of a generic spin Hamiltonian in a stroboscopic way when subjected to appropriate laser pulses, and controlled displacements, which allow us to implement the single-site and two-sites contributions to the Hamiltonian. The key point is that in this case the sequences of gates allows us to discriminate between the system and the bath. A detailed account of the pulse sequence needed to realize stroboscopically the spin chain and its coupling to the two-level system will be presented elsewhere [18]. The types of baths that one can simulate by these means embrace Ising, XY and Heisenberg exchange Hamiltonian. Therefore by varying the parameters of the optical lattice we can test the impact of the different phases (critical, ferromagnetic, anti-ferromagnetic, etc ...) of the environment on the decoherence of the two-level system.

Additional motivation to analyse these models stems from the great interest in the decoherence due to spin baths both in the absence [19, 20] and in the presence [16, 21–27] of interaction among the spins of the bath. The scenario we consider here (see the next section), however, has a crucial difference from those discussed so far. It goes beyond the model of a central spin coupled uniformly with all the spins of the bath and therefore it can be tested experimentally. The difference is not merely quantitative but qualitative.

3. The model

We consider a qubit coupled to a one-dimensional array of spin-1/2 particles. The composite system-plus-bath Hamiltonian is given by

$$\mathcal{H} = \mathcal{H}_{\mathrm{TL}} + \mathcal{H}_{\mathrm{E}} + \mathcal{H}_{\mathrm{IN}}. \tag{1}$$

The two-level (TL) system is characterized by the free Hamiltonian $\mathcal{H}_{\mathrm{TL}} = \omega\tau_z$, where τ_α denote the Pauli matrices representing the qubit. The interaction (IN) between the system and the bath is described by: $\mathcal{H}_{\mathrm{IN}} = -\epsilon\tau_z\sigma_1^z$, with ϵ being the coupling constant. The Hamiltonian of the environment (E) is:

$$\mathcal{H}_{\mathrm{E}} = -\frac{J}{2}\sum_{j=1}^{N}\left[(1+\gamma)\sigma_j^x\sigma_{j+1}^x + (1-\gamma)\sigma_j^y\sigma_{j+1}^y + \Delta\sigma_j^z\sigma_{j+1}^z + 2\lambda\sigma_j^z\right], \tag{2}$$

where σ_i^α ($\alpha = x, y, z$) are the Pauli matrices of the ith spin of the bath. The constants J, Δ, γ and λ represent the exchange coupling, the anisotropy parameter along z and in the

[6] It would be quite interesting to consider as an engineered bath a 3D optical lattice. Besides being feasible from an experimental point of view, this could be useful in studying for instance the situation found in solid-state NMR [17]. It would be also intriguing to study the Bose–Hubbard model as a bath, which would make the experimental realization even simpler. We consider 1D baths as, in several cases, are amenable of an exact solution.

xy plane respectively, and an external magnetic field. The model defined by equation (2) has a very rich structure [28]: when the anisotropy parameter Δ is set to zero, for all the interval $0 < \gamma \leqslant 1$ it belongs to the Ising universality class; at the thermodynamic limit it has a critical point at $\lambda = \lambda_c = 1$. For $\gamma = 0$ it reduces to the isotropic XY model, which is critical for $|\lambda| \leqslant 1$. In the case when there is a non zero z anisotropy and no magnetic field applied (i.e. $\Delta \neq 0, \lambda = 0$), the model reduces to Heisenberg XXZ. It is critical for $-1 \leqslant \Delta \leqslant 1$, in the other cases the chain has ferromagnetic or anti-ferromagnetic order if the anisotropy is positive or negative, respectively. In the following we will only consider situations without z anisotropy: $\Delta = 0$ (the case $\Delta \neq 0$ will be discussed in a separate publication [18]). We will also suppose that the bath is initially set up in its ground state, and use periodic boundary conditions.

With this choice of $\mathcal{H}_{\mathrm{TL}}$ and $\mathcal{H}_{\mathrm{IN}}$, the evolution of the reduced density matrix ρ of the two-level system corresponds to a purely dephasing process [29, 30]: in the basis of the eigenstates of $\tau_z\{|\uparrow\rangle, |\downarrow\rangle\}$, the diagonal terms $\rho_{\uparrow\uparrow}(t)$ and $\rho_{\downarrow\downarrow}(t)$ do not evolve in time, since $[\tau_z, \mathcal{H}] = 0$. Only the off-diagonal terms will decay according to the expression $\rho_{\uparrow\downarrow}(t) = \rho_{\uparrow\downarrow}(0)\,\mathrm{e}^{-\mathrm{i}\omega t} D(t)$. The decoherence of the system can then be captured by the real function

$$\mathcal{L}(t) = |D(t)|^2 = |\langle \mathrm{e}^{\mathrm{i}\mathcal{H}_\downarrow t}\,\mathrm{e}^{-\mathrm{i}\mathcal{H}_\uparrow t}\rangle|^2, \tag{3}$$

sometimes called Loschmidt echo, where $\mathcal{H}_{\uparrow/\downarrow} = \mathcal{H}_E \mp \epsilon\sigma_1^z$; the average is evaluated over the ground state of the spin bath. Values of $\mathcal{L}(t)$ close to 1 indicate a weak coupling ϵ between the system and the environment, while $\mathcal{L}(t) \sim 0$ corresponds to the opposite case of strong coherence suppression due to the interaction with the bath.

We now proceed by considering $\Delta = 0$; this case is particularly relevant from a theoretical point of view, since the function $\mathcal{L}(t)$ can be calculated exactly. By means of the Jordan–Wigner transformation $\sigma_j^+ = c_j^\dagger \exp\left(\mathrm{i}\pi \sum_{k=1}^{j-1} c_k^\dagger c_k\right)$, and $\sigma_j^z = 2c_j^\dagger c_j - 1$, it is possible to map the Hamiltonian of the spin bath plus interaction onto a free fermion model [31], which can be expressed in the following quadratic form:

$$\mathcal{H}_{\uparrow/\downarrow} = \tfrac{1}{2}\Psi^\dagger \mathbf{C}\Psi, \tag{4}$$

where $\Psi^\dagger = \left(c_1^\dagger \ldots c_N^\dagger c_1 \ldots c_N\right)$ (c_i are the corresponding spinless fermion operators) and $\mathbf{C} = \sigma^z \otimes \mathbf{A} + \mathrm{i}\sigma^y \otimes \mathbf{B}$ is a tridiagonal block matrix with

$$A_{j,k} = -J(\delta_{k,j+1} + \delta_{j,k+1}) - 2(\lambda \pm \epsilon\delta_{j,1})\delta_{j,k} \tag{5}$$

$$B_{j,k} = -J\gamma(\delta_{k,j+1} - \delta_{j,k+1}). \tag{6}$$

The Loschmidt echo can then be evaluated exactly [32], leading to the expression

$$\mathcal{L}(t) = \det(1 - \mathbf{r} + \mathbf{r}\,\mathrm{e}^{\mathrm{i}\mathbf{C}t}), \tag{7}$$

where $\mathbf{r}$ is a matrix whose elements $r_{i,j} = \langle \Psi_i^\dagger \Psi_j \rangle$ are the two-point correlation functions of the spin chain. This is one of the central results of this work. Equation (7) allows us to go beyond the central spin model considered so far in the literature and enables us to explicitly address the case of a large number of spins in the bath, since it provides an explicit formula for the Loschmidt echo in terms of a determinant of a $2N \times 2N$ matrix. In the following we will study the dependence of $\mathcal{L}(t)$ upon the spin bath Hamiltonian for a number N of bath spins of the order of $\sim 10^2$–10^3.

4. Decay of the Loschmidt echo

We first focus on the case $\gamma = 1$ in equation (2), that is when the environment is modelled by an Ising chain in a transverse magnetic field. Figure 2(*a*) shows the generic behaviour of $\mathcal{L}$ as

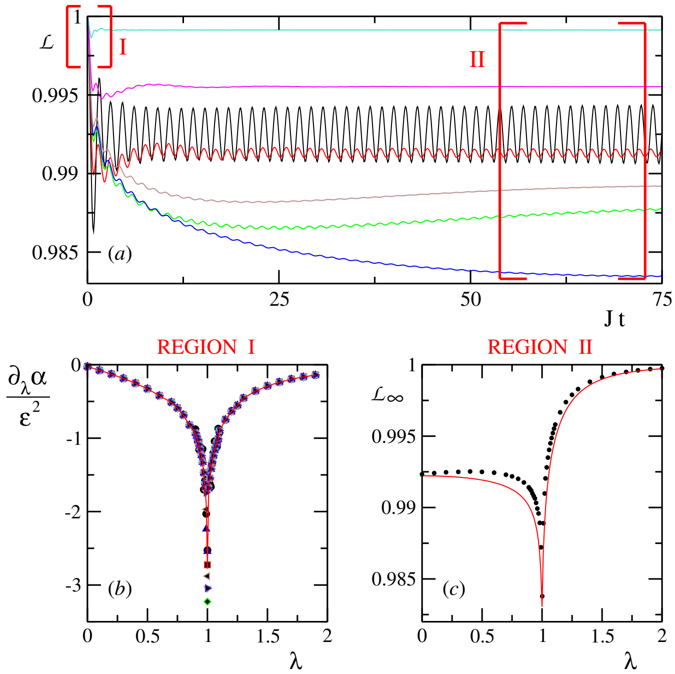

Figure 2. (a) Loschmidt echo as a function of time for a qubit coupled to a $N = 300$ spin Ising chain; $\epsilon = 0.125$. The various curves are for different values of the transverse magnetic field: $\lambda = 0.5$ (black), 0.9 (red), 0.99 (green), 1 (blue), 1.01 (brown), 1.1 (magenta), 1.5 (cyan). (b) In the region I, $\mathcal{L}$ has a Gaussian decay with a typical scale α. Here we plot the derivative $\epsilon^{-2}\partial_\lambda\alpha$ as a function of λ in order to highlight the change in the decoherence process when the chain undergoes a phase transition. The different values of ϵ (0.005 (circles), 0.0125 (squares), 0.025 (diamonds), 0.0375 (triangles up), 0.05 (triangles left), 0.125 (triangles down), 0.25 (triangles right)) are nevertheless all in the perturbative regime so that the curves overlap when rescaled by the factor ϵ^2. The solid line shows the result of a perturbative calculation at small times. (c) In the region II, $\mathcal{L}$ oscillates around a constant value $\mathcal{L}_\infty$. Here we plot $\mathcal{L}_\infty$ as a function of λ for $\epsilon = 0.125$. Points indicate the data from the exact solution, while the solid line is the result of the perturbation expansion in ϵ, equation (10).

a function of time for different values of λ, and fixed coupling constant ϵ. For $\lambda < 1$ the echo oscillates with a frequency proportional to ϵ, while for $\lambda > 1$ the amplitude of oscillations is drastically reduced. The Loschmidt echo reaches its minimum value at the critical point $\lambda_c = 1$, thus revealing that the decoherence is enhanced by the criticality of the environment. Since the chain is finite, at long times there are revivals, but for $N \sim 10^3$ there is already a wide interval (region II) where the asymptotic behaviour can be analysed. A detailed analysis of the short- and long-time behaviour (regions I and II in figure 2(a)) of $\mathcal{L}$ reveals a number of interesting features.

4.1. Short-time behaviour

At small times the decay is Gaussian:

$$\mathcal{L}_I(t) \sim \mathrm{e}^{-\alpha t^2}. \tag{8}$$

This behaviour can be predicted within a second order time perturbation theory in the coupling ϵ between the system and the bath. The scale of the Gaussian decay at short times displays a remarkable universal behaviour. The very existence of the critical point appears in the *slope* of α, shown in figure 2(*b*). By taking the derivative with respect to λ, $\partial_\lambda \alpha$ shows a logarithmic singularity, characteristic of the Ising model, $\partial_\lambda \alpha \sim |\log(\lambda - \lambda_c)|$.

4.2. Long-time behaviour

At long times (region II) and for $\lambda > 1$ the Loschmidt echo approaches an asymptotic value $\mathcal{L}_\infty$, while for $\lambda < 1$ it oscillates around a value which is constant in time (see figure 2(*a*) for a qualitative picture). Also this limiting value $\mathcal{L}_\infty$ has clear signatures of the critical behaviour of the spin environment as shown in figure 2(*c*), since it manifests a cusp at the critical point. Evidence that $\mathcal{L}_\infty$ describes the asymptotic regime can be obtained by comparing data with the result of an analytical expression based on the following simple ansatz: we assume that, as the bath approaches the thermodynamic limit $N \to \infty$, the coherence loss saturates and it is constant after a given transient time t_0:

$$\mathcal{L} \approx \mathcal{L}_\infty \qquad \forall\, t \geqslant t_0. \tag{9}$$

After expanding the ground state of the spin bath onto the eigenbasis of $\mathcal{H}_\uparrow$, from the above ansatz we found that a rough estimate of the limit value $\mathcal{L}_\infty$ is given by the overlap between the ground states of the two coupled Hamiltonians $\mathcal{H}_\uparrow$ and $\mathcal{H}_\downarrow$. A second-order perturbation expansion in the coupling ϵ eventually gives

$$\mathcal{L}_\infty \approx \left[1 - \frac{\epsilon^2}{2} \sum_{k \neq 0} \frac{\left|\langle \psi_k | \sigma_1^z | \psi_0 \rangle\right|^2}{(E_k - E_0)^2} \right]^4, \tag{10}$$

where $|\psi_{k/0}\rangle$ are the excited states (ground state) of $\mathcal{H}_E$ with energy $E_{k/0}$.

4.3. Scaling with the bath size

Additional interesting information emerge by analysing the scaling of $\mathcal{L}$ with the size N of the bath. If the chain is not at the critical point and N is large, both the α and the saturation value $\mathcal{L}_\infty$ are almost independent of the bath size. At the critical point the situation is rather peculiar. Due to the infinite correlation length, the decay is very slow in time $\mathcal{L}^c(t) \sim \ln^{-1} t$, and the minimum value $\mathcal{L}^c_\infty$ reached by the Loschmidt echo depends on N as

$$\mathcal{L}^c_\infty = \frac{l_\infty}{1 + \beta \ln N}. \tag{11}$$

This is shown in figure 3, where we plot the decay of the Loschmidt echo in time at λ_c, for different N and fixed perturbation strength ϵ. In the inset we report the minima of $\mathcal{L}^c(t)$ as a function of N; dotted-dashed line is a numerical fit of data, of the form in equation (11). We expect that the coherence loss should go to zero at the thermodynamic limit $N \to \infty$; nonetheless this is hard to see numerically, since the decay is logarithmic, and the actual value of $\mathcal{L}^c_0$ is still very far from zero, even for $N = 2000$ spins.

A quantum critical bath manifests itself in a slow decay of the Loschmidt echo and in an anomalous behaviour of its asymptotic value. These results rely only on the existence of an infinite correlation length and therefore are valid for any model of spin baths. On the opposite the behaviour of $\mathcal{L}$ is very sensitive to the number of spins of the bath to which the two-level system is coupled if the chain is critical. The differences between our model and the central spin model are not merely quantitative. If however the chain is non-critical, these differences can be included in a renormalization of the effective coupling strength.

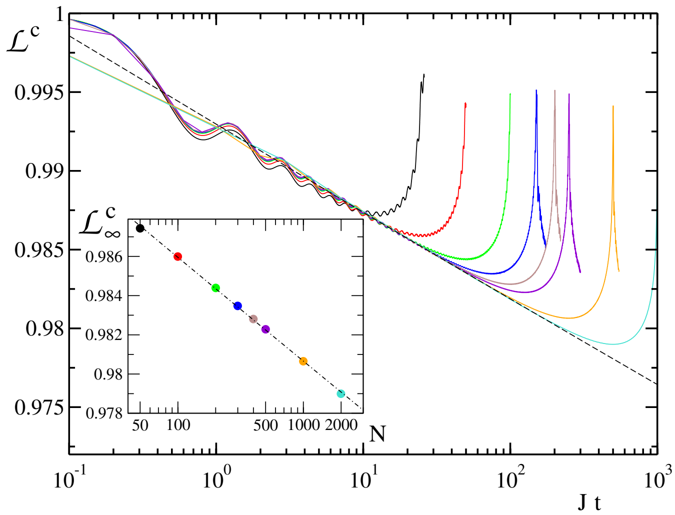

Figure 3. Loschmidt echo as a function of time at criticality ($\lambda = 1$), for different sizes of the chain: $N = 50$ (black), 100 (red), 200 (green), 300 (blue), 400 (brown), 500 (violet), 1000 (orange), 2000 (cyan). The perturbation strength is kept fixed: $\epsilon = 0.125$. Dashed line is a guideline that shows a decay of type: $\mathcal{L}^c(t) = c_0/(1 + c_1 \ln t)$. Inset: minimum value of $\mathcal{L}^c$ as a function of N. Numerical data have been fitted with $\mathcal{L}^c_\infty = l_\infty/(1 + \beta \ln N)$ (dotted-dashed line), where $l_\infty \approx 0.996\,71$, $\beta \approx 2.369\,33 \times 10^{-3}$.

4.4. The XY spin bath

All the results presented in this section until now concern the case $\gamma = 1$ in equation (2). Nonetheless the properties of the Loschmidt echo described so far are typical as long as $0 < \gamma \leqslant 1$, that is when the spin bath belongs to the Ising universality class, the critical point being in $\lambda_c = 1$. Instead, if the chain is described by the isotropic XY model ($\gamma = 0$), the situation dramatically changes. In this case indeed the environment does not belong to the Ising universality class: it exhibits a critical behaviour for all the parametric range $|\lambda| \leqslant 1$, while it is ferromagnetic (anti-ferromagnetic) for $\lambda > 1$ ($\lambda < 1$). The decay of the Loschmidt echo follows this classification, since it behaves as in equation (11) for all the region $|\lambda| < 1$. In the ferromagnetic case $\lambda > 1$ instead we found $\mathcal{L}(t) = 1$; the environment is frozen in the perfect ferromagnetic ground state, therefore the coupled qubit does not decohere at all.

5. Conclusions

In this work we proposed the use of optical lattices as open quantum system simulators. This approach, in our opinion, may lead to several interesting advantages in understanding decoherence in quantum systems. The key point of our proposal is that it should be possible to experimentally test the time decay of coherences in a fully *controllable* and *tunable* environment. The class of baths that can be simulated in this fashion is rather rich and embraces several distinct classes of universality for spin chains. Here, as an example, we presented the results for an Ising bath which turns out to be exactly solvable. Once more we stress that our model differs in a substantial manner from the central spin model discussed so far in the literature, this last being very hard to implement in the laboratory.

There are several issues that we are currently addressing and will be subject of a forthcoming publication [18]. We quote for example the connection between decoherence and entanglement, the case with $\Delta \neq 0$, the crossover to the central spin model. We also mention that spin baths which are simulatable within our framework do not include generic non-integrable models. These could show new and different physics [33], even though their experimental implementation may be far from being simply feasible, and numerical studies much more limited.

Acknowledgments

We thank D Cory for very useful discussions. We acknowledge support from EC (RTNNANO, SPINTRONICS, ACQP, SCALA and EUROSQIP), MIUR-PRIN, NSF (grant of the ITAMP at Harvard University and Smithsonian Astrophysical Observatory) and CRM 'E De Giorgi' of SNS.

References

[1] Lewenstein M, Sanpera A, Ahufinger V, Damski B, Sen(De) A and Sen U 2006 *Preprint* cond-mat/0606771
[2] Greiner M, Mandel O, Esslinger T, Hänsch T W and Bloch I 2002 *Nature* **415** 39
[3] Jaksch D, Bruder C, Cirac J I, Gardiner C W and Zoller P 1998 *Phys. Rev. Lett.* **81** 3108
[4] Duan L-M, Demler E and Lukin M D 2003 *Phys. Rev. Lett.* **91** 090402
[5] Jané E, Vidal G, Dür W, Zoller P and Cirac J I 2003 *Quantum Inf. Comp.* **3** 15
[6] Zurek W H 1991 *Phys. Today* **44** 36
[7] Zurek W H 2003 *Rev. Mod. Phys.* **75** 715
[8] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[9] Weiss U 1999 *Quantum Dissipative Systems* 2nd edn (Singapore: World Scientific)
[10] Prokof'ev N V and Stamp P C E 2000 *Rep. Prog. Phys.* **63** 669
[11] Peres A 1984 *Phys. Rev.* A **30** 1610
[12] Jalabert R A and Pastawski H M 2001 *Phys. Rev. Lett.* **86** 2490
[13] Cucchietti F M, Dalvit D A R and Zurek W H 2003 *Phys. Rev. Lett.* **91** 210403
[14] Znidaric M and Prosen T 2003 *J. Phys. A: Math. Gen.* **36** 2463
[15] Gorin T, Prosen T, Seligman T H and Strunz W T 2004 *Phys. Rev.* A **70** 042105
[16] Quan H T, Song Z, Liu X F, Zanardi P and Sun C P 2006 *Phys. Rev. Lett.* **96** 140604
[17] Cory D G *et al* 2000 *For. der Phys.* **48** 875
[18] Rossini D, Calarco T, Giovannetti V, Montangero S and Fazio R 2006 *Preprint* quant-ph/0611242
[19] Zurek W H 1982 *Phys. Rev.* D **26** 1862
[20] Cucchietti F M, Paz J P and Zurek W H 2005 *Phys. Rev.* A **72** 052113
[21] Tessieri L and Wilkie J 2003 *J. Phys. A: Math. Gen.* **36** 12305
[22] Dawson C M, Hines A P, McKenzie R H and Milburn G J 2005 *Phys. Rev.* A **71** 052321
[23] Paganelli S, Pasquale F de and Giampaolo S M 2002 *Phys. Rev.* A **66** 052317
[24] Cucchietti F M, Fernandez-Vidal S and Paz J P 2006 *Preprint* quant-ph/0604136
[25] Ou Y-C and Fan H 2006 *Preprint* quant-ph/0606053
[26] Khveshchenko D V 2003 *Phys. Rev.* B **68** 193307
[27] Dobrovitski V V, Katsnelson M I and Harmon B N 2003 *Phys. Rev. Lett.* **90** 067201
[28] Sachdev S 2000 *Quantum Phase Transitions* (Cambridge: Cambridge University Press)
[29] Unruh W G 1995 *Phys. Rev.* A **51** 992
[30] Palma G M, Suominem K A and Ekert A K 1996 *Proc. R. Soc.* A **452** 567
[31] Lieb E, Schultz T and Mattis D 1961 *Ann. Phys.* **16** 407
[32] Levitov L S, Lee H and Lesovik G B 1996 *J. Math. Phys.* **37** 4845
Klich I 2003 *Quantum Noise in Mesoscopic Physics* ed Yu V Nazarov *(NATO Science Series*, vol 97*)* (Dordrecht: Kluwer) (*Preprint* cond-mat/0209642)
[33] Prosen T 2000 *Prog. Theor. Phys. Suppl.* **139** 191
Prosen T 2002 *Phys. Rev.* E **65** 036208